For example,

$$(2 \times 10^6) + (9 \times 10^{-3}) = 2{,}000{,}000 + 0.009$$
$$= 2{,}000{,}000.009 \approx 2 \times 10^6$$

where the symbol $\approx$ means "is approximately equal to."

When raising a power to another power, the exponents are multiplied. For example,

$$(10^2)^4 = 10^2 \times 10^2 \times 10^2 \times 10^2 = 10^8$$

1-5 Significant Figures and Order of Magnitude

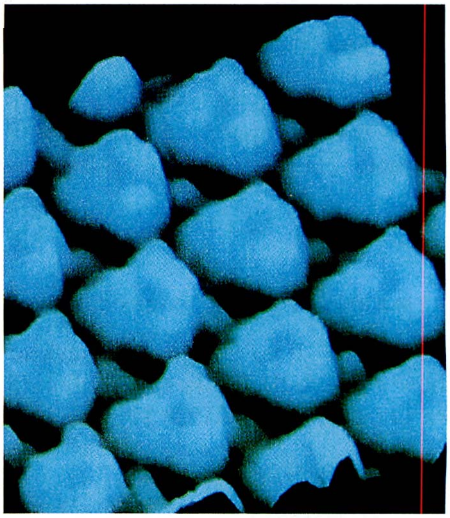
Many of the numbers in science are the result of measurement and are therefore known only to within some degree of experimental uncertainty. The magnitude of the uncertainty depends on the skill of the experimenter and the apparatus used, and often can only be estimated. A rough indication of the uncertainty in a measurement is inferred by the number of digits used. For example, if we say that a table is 2.50 m long, we are often saying that its length is between 2.495 m and 2.505 m. That is, we know the length to about $\pm$ 0.005 m = $\pm$ 0.5 cm. If we used a meterstick with millimeter markings and measured the table length carefully, we might estimate that we could measure the length to $\pm$ 0.5 mm rather than $\pm$ 0.5 cm. We would indicate this precision when giving the length by using four digits, such as 2.503 m. A reliably known digit (other than a zero used to locate the decimal point) is called a **significant figure.** The number 2.50 has three significant figures; 2.503 m has four. The number 0.001 03 has three significant figures. (The first three zeroes are not significant figures but merely locate the decimal point.) In scientific notation, the number 0.001 03 is written 1.03×10^{-3}. A common student error is to carry more digits than the certainty of measurement warrants. Suppose, for example, that you measure the area of a circular playing field by pacing off the radius and using the formula for the area of a circle, $A = \pi r^2$. If you estimate the radius to be 8 m and use a 10-digit calculator to compute the area, you obtain $\pi(8\text{ m})^2 = 201.0619298\text{ m}^2$. The digits after the decimal point give a false indication of the accuracy with which you know the area. If you found the radius by pacing, you might expect that your measurement was accurate to only about 0.5 m. That is, the radius could be as great as 8.5 m or as small as 7.5 m. If the radius is 8.5 m, the area is $\pi(8.5\text{ m})^2 = 226.9800692\text{ m}^2$, whereas if it is 7.5 m, the area is $\pi(7.5\text{ m})^2 = 176.714587\text{ m}^2$. There is a general rule to guide you when combining several numbers in multiplication or division:

> The number of significant figures in the result of multiplication or division is no greater than the least number of significant figures in any of the factors.

In the previous example, the radius is known to only one significant figure, so the area is also known only to one significant figure. It should be written as $2 \times 10^2\text{ m}^2$, which says that the area is probably between 150 m^2 and 250 m^2.

The precision of the sum or difference of two measurements is only as good as the precision of the least precise of the two measurements. A general rule is:

> The result of addition or subtraction of two numbers has no significant figures beyond the last decimal place where both of the original numbers had significant figures.



Benzene molecules of the order of 10^{-10} m in diameter as seen in a scanning electron microscope.

Chromosomes measuring on the order of 10^{-6} m across as seen in a scanning electron microscope.

SIGNIFICANT FIGURES

EXAMPLE 1-5

Find the sum of 1.040 and 0.21342.

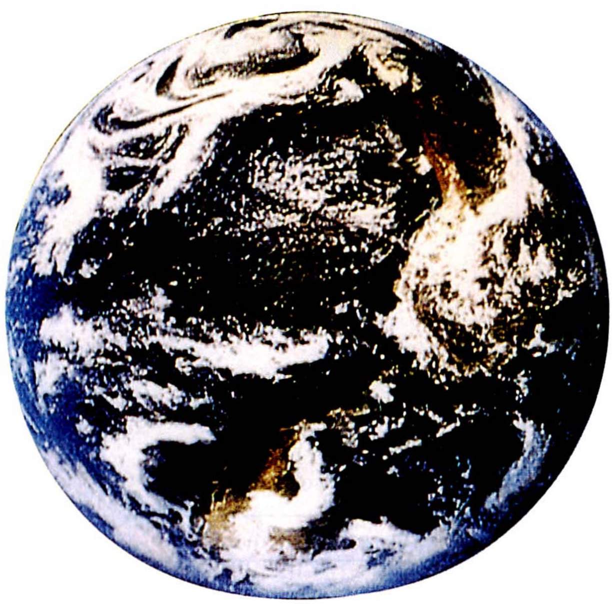
PICTURE THE PROBLEM The first number, 1.040, has only three significant figures beyond the decimal point, whereas the second, 0.21342 has five. According to the rule stated above, the sum can have only three significant figures beyond the decimal point.

Sum the numbers, keeping only three digits beyond the decimal point: $1.040 + 0.21342 = \boxed{1.253}$

EXERCISE Apply the appropriate rule for significant figures to calculate the following: (*a*) 1.58×0.03, (*b*) $1.4 + 2.53$, (*c*) $2.34 \times 10^2 + 4.93$ (*Answer* (*a*) 0.05, (*b*) 3.9, (*c*) 2.39×10^2)

Most examples and exercises in this book will be done with data to three (or sometimes four) significant figures, but occasionally we will say, for example, that a table top is 3 ft by 8 ft rather than taking the time and space to say it is 3.00 ft by 8.00 ft. Any data you see in an example or exercise can be assumed to be known to three significant figures unless otherwise indicated. The same assumption holds for the data in the end-of-chapter problems. In doing rough calculations or comparisons, we sometimes round off a number to the nearest power of 10. Such a number is called an **order of magnitude.** For example, the height of a small insect, say an ant, might be 8×10^{-4} m $\approx 10^{-3}$ m. We would say that the order of magnitude of the height of an ant is 10^{-3} m. Similarly, though the height of most people is about 2 m, we might round that off and say that the order of magnitude of the height of a person is 10^0 m. By this we do not mean to imply that a typical height is really 1 m but that it is closer to 1 m than to 10 m or to $10^{-1} = 0.1$ m. We might say that a typical human being is three orders of magnitude taller than a typical ant, meaning that the ratio of heights is about 1000 to 1. An order of magnitude does not provide any digits that are reliably known.

Distances familiar in our everyday world. The height of the woman is of the order of 10^0 m and that of the mountain is of the order of 10^4 m.



Earth, with a diameter of the order of 10^7 m, as seen from space.

The diameter of the Andromeda galaxy is of the order of 10^{21} m.

TABLE 1-3

The Universe by Orders of Magnitude

Size or Distance	(m)
Proton	10^{-15}
Atom	10^{-10}
Virus	10^{-7}
Giant amoeba	10^{-4}
Walnut	10^{-2}
Human being	10^{0}
Highest mountain	10^{4}
Earth	10^{7}
Sun	10^{9}
Distance from earth to sun	10^{11}
Solar system	10^{13}
Distance to nearest star	10^{16}
Milky Way galaxy	10^{21}
Visible universe	10^{26}

Mass	(kg)
Electron	10^{-30}
Proton	10^{-27}
Amino acid	10^{-25}
Hemoglobin	10^{-22}
Flu virus	10^{-19}
Giant amoeba	10^{-8}
Raindrop	10^{-6}
Ant	10^{-4}
Human being	10^{2}
Saturn V rocket	10^{6}
Pyramid	10^{10}
Earth	10^{24}
Sun	10^{30}
Milky Way galaxy	10^{41}
Universe	10^{52}

Time Interval	(s)
Time for light to cross nucleus	10^{-23}
Period of visible light radiation	10^{-15}
Period of microwaves	10^{-10}
Half-life of muon	10^{-6}
Period of highest audible sound	10^{-4}
Period of human heartbeat	10^{0}
Half-life of free neutron	10^{3}
Period of earth's rotation	10^{5}
Period of earth's revolution around sun	10^{7}
Lifetime of human being	10^{9}
Half-life of plutonium-239	10^{12}
Lifetime of mountain range	10^{15}
Age of earth	10^{17}
Age of universe	10^{18}

It may be thought of as having no significant figures. Table 1-3 gives some typical order-of-magnitude values for a variety of sizes, masses, and time intervals encountered in physics.

In many cases the order of magnitude of a quantity can be estimated using reasonable assumptions and simple calculations. The physicist Enrico Fermi was a master at using cunning order-of-magnitude estimations to generate answers for questions that seemed impossible to calculate because of lack of information. Problems like these are often called **Fermi questions.** The following is an example of a Fermi question.

EXPLORING

How many piano tuners are there in Chicago? Find out this, and more, at www.whfreeman.com/tipler5e.

BURNING RUBBER — **EXAMPLE 1-6**

What thickness of rubber tread is worn off the tire of an automobile as it travels 1 km (0.6 mi)?

PICTURE THE PROBLEM We assume the tread thickness of a new tire is 1 cm. This may be off by a factor of two or so, but 1 mm is certainly too small and 10 cm is too large. Since tires have to be replaced after about 60,000 km (about 37,000 mi), we will assume that the tread is completely worn off after 60,000 km. In other words, the rate of wear is 1 cm of tire per 60,000 km of travel.

Use 1 cm wear per 60,000 km travel to compute the thickness worn after 1 km of travel:

$$\frac{1\text{ cm wear}}{60{,}000\text{ km travel}} = \frac{1.7\times10^{-5}\text{ cm wear}}{1\text{ km travel}}$$

$$\boxed{\approx 0.2\ \mu\text{m wear per km of travel}}$$

EXERCISE How many grains of sand are on a 0.50 km stretch of beach that is 100 m wide? *Hint: Assume that the sand is* 3 m *deep. Estimate that the diameter of one grain of sand is* 1.00 mm. (*Answer* $\approx 2\times10^{14}$)

SUMMARY

The fundamental units in the SI system are the meter (m), the second (s), the kilogram (kg), the kelvin (K), the ampere (A), the mole (mol), and the candela (cd). The unit(s) of every physical quantity can be expressed in terms of these fundamental units.

Topic	Relevant Equations and Remarks
1. **Units**	The magnitude of physical quantities (for example, length, time, force, and energy) are expressed as a number times a unit.
Fundamental units	The fundamental units in the SI system (short for *Système International*) are the meter (m), the second (s), the kilogram (kg), the kelvin (K), the ampere (A), the mole (mol), and the candela (cd). The unit(s) of every physical quantity can be expressed in terms of these fundamental units.
Units in equations	Units in equations are treated just like any other algebraic quantity.
Conversion	Conversion factors, which are always equal to 1, provide a convenient method for converting from one kind of unit to another.
2. **Dimensions**	The two sides of an equation must have the same dimensions.
3. **Scientific Notation**	For convenience, very small and very large numbers are generally written as a factor times a power of 10.
4. **Exponents**	
Multiplication	When multiplying two numbers, the exponents are added.
Division	When dividing two numbers, the exponents are subtracted.
Raising to a power	When a number containing an exponent is itself raised to a power, the exponents are multiplied.
5. **Significant Figures**	
Multiplication and division	The number of significant figures in the result of multiplication or division is *no greater than* the least number of significant figures in any of the numbers.
Addition and subtraction	The result of addition or subtraction of two numbers has no significant figures beyond the last decimal place where both of the original numbers had significant figures.
6. **Order of Magnitude**	A number rounded to the nearest power of 10 is called an order of magnitude. The order of magnitude of a quantity can often be estimated using reasonable assumptions and simple calculations.